UH-6221

B. E. - II (Sem. III) (IC) Examination

May / June – 2012

Engineering Mathematics : Paper - III

(Old Course)

Time : **3** Hours] [Total Marks : **100**

Instructions :

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ B. E. - 2 (SEM. 3) (IC)

Name of the Subject :
☛ ENGINEERING MATHEMATICS - 3 (OLD)

☛ Subject Code No. : 6 2 2 1 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures on right indicate full marks.

1 (a) Answer the following : **10**

(1) Find grad ϕ when ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, -1)$.

(2) Find $div \vec{V}$ where

$$\vec{V} = \left(x^2 + yz\right)\hat{i} + \left(y^2 + zx\right)\hat{j} + \left(z^2 + xy\right)\hat{k}$$

(3) Evaluate the integral $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \, dxdydz$

(4) Write the Fourier coefficients for the function $f(x)$ defined in the interval $(c, c+2\pi)$.

(5) What are even functions ? Give the Euler's formulae for an even function defined in $(-\pi, \pi)$.

(b) Attempt any **four** : **12**

(1) Evaluate $\iint_R (4-x-y)\,dxdy$ over the region $R = 0 \le x \le 2,\ \ 0 \le y \le 1$.

(2) Evaluate $\int_0^1 \int_{4y}^4 e^{x^2}\,dx\,dy$ by changing the order of integration.

(3) Change into polar coordinates and evaluate $\iint_R \left(x^2+y^2\right) x\,dxdy$ over the positive quadrant of the circle $x^2+y^2=a^2$.

(4) Calculate, by double integration, the volume generated by the revolution of the cardioid $r = a(1-\cos\theta)$ about its axis.

(5) Find the mass of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, the variable density $\rho = kxyz$.

2 (a) Attempt any **two** : **6**

(1) A particle moves along the curve $x=t^2+1,\ y=t^2$, $z=2t+5$ where t is the time. Find the components of its velocity and acceleration at $t=1$ in the direction $\hat{i}+\hat{j}+3\hat{k}$.

(2) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $p(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$.

(3) Define an irrotational vector. Show that the vectors field $\vec{V} = 2xyz^3 \hat{i} + x^2 z^3 \hat{j} + 3x^2 yz^2 \hat{k}$ is irrotational.

(b) Attempt any **two** : **8**

(1) If $\vec{F} = 3xy \hat{i} - y^2 \hat{j}$, evaluate $\int_C \vec{F} \cdot \vec{dr}$ where C is the arc of the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.

(2) Verify Green's theorem in the plane for $\oint_C \left[\left(3x^2 - 8y^2\right)dx + (4y - 6xy)dy\right]$ where C is the boundary of the region defined by $x = 0$, $y = 0$, $x + y = 1$.

(3) Evaluate the surface integral $\iint_S curl\, \vec{F} \cdot \hat{n}\, dS$ by transforming it into a line integral, S being that part of the surface of the paraboloid $z = 1 - x^2 - y^2$, for which $z \geq 0$ and $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$.

3 (a) Expand $f(x)=x$ as a half-range cosine series in $0<x<2$. **4**

(b) Attempt any two : **10**

(1) Obtain the Fourier series to represent e^x in the interval $0<x<2\pi$.

(2) Find the Fourier series to represent $f(x)=x^2-2$ when $-2 \le x \le 2$.

(3) Find the Fourier series expansion for $f(x)$ if

$$f(x)=\begin{cases} -\pi & , \quad -\pi<x<0 \\ x & , \quad 0<x<\pi \end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots\ldots=\frac{\pi^2}{8}$

4 (a) Do as directed : **10**

(1) Find the value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$.

(2) Define error function and write the value of $erf(0)$.

(3) Find the inverse Laplace transform of

$$\bar{f}(S)=\frac{1}{(S-1)^2+4}.$$

(4) Define critical point of the function $w=f(z)$ and obtain critical point of the function $w=z^2-1$.

(5) Show that for the analytic function $w=f(z)=u+iv$, u and v are harmonic functions.

(b) Attempt any **two** : **6**

(1) Show that $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1} dy = \frac{\Gamma(p)}{q^p}$, where $p>0, q>0$.

(2) Prove that $\beta(m,n)=\beta(m+1,n)+\beta(m,n+1)$

(3) Show that $\frac{d}{dx}\left[erf(ax)\right]=\frac{2a}{\sqrt{x}}e^{-a^2x^2}$.

(c) Attempt any **two** : **6**

(1) $pz-qz=z^2+(x+y)^2$

(2) $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

(3) $p\tan x+q\tan y=\tan z$.

5 (a) Attempt any **one** : **6**

(1) Find the deflection $y(x,t)$ of the vibrating string of length π and ends fixed, corresponding to zero initial velocity and initial deflection $f(x)=k(\sin x-\sin 2x)$, given $C^2=1$.

(2) Show that the solution of the differential equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$, subject to the conditions,

(i) u not infinite for $t\to\infty$,

(ii) $\frac{\partial u}{\partial x}=0$ for $x=0$ and $x=\ell$,

(iii) $u=\ell x-x^2$ for $t=0$, between $x=0$ and $x=\ell$,

is $u=\frac{1}{6}\ell^2-\frac{\ell^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\frac{2n\pi x}{\ell}e^{-4n^2\pi^2kt/\ell^2}$.

(b) Attempt any **two** : 8

(1) Find the Laplace transform of $f(t) = e^{-3t}(2\cos 5t - 3\sin 5t)$.

(2) Find the inverse Laplace transform of $\frac{S^2}{(S-2)^3}$.

(3) Solve the equation

$y'' - 3y' + 2y = 4t + e^{3t}$, when $y(0) = 1$ and $y'(0) = -1$.

6 (a) Attempt any **two** : 8

(1) If $f(z)$ is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

(2) Consider the transformation $\omega = ze^{i\pi/4}$ and determine the region in the ω-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane.

(3) Find the image of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $\omega = \frac{1}{z}$. Also show the regions graphically.

(b) Attempt any two : 6

(1) Evaluate $\oint_C \frac{e^{2z}}{(z-1)(z-3)} dz$, where C is the circle $|z| = 2$.

(2) Evaluate $\int_{0}^{4+2i} \bar{z}\, dz$, along the curve given by $z = t^2 + it$.

(3) Evaluate $\oint_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$, where C is the circle $|z| = 1$.
